Hydromagnetic dynamos in rotating spherical fluid shells in dependence on the Prandtl number and stratification

Ján Šimkanin and Pavel Hejda

Institute of Geophysics, Academy of Sciences of CR, Prague, Czech Republic
Parameters

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Core</th>
<th>Dynamo models</th>
</tr>
</thead>
<tbody>
<tr>
<td>$R_a/R_{ac}$</td>
<td>$\gg 1 \left( R_a \sim 10^{30} \right)$</td>
<td>$1 - 50$</td>
</tr>
<tr>
<td>$E$</td>
<td>$\mathcal{O}(10^{-15})$</td>
<td>$\mathcal{O}(10^{-7})$</td>
</tr>
<tr>
<td>$P_m$</td>
<td>$\mathcal{O}(10^{-7})$</td>
<td>$0.01 - 1$</td>
</tr>
<tr>
<td>$P_r$</td>
<td>0.2</td>
<td>0.1 - 1</td>
</tr>
</tbody>
</table>

$P_{m_{min}} \approx 450 E^{3/4}$

Christensen, Olson and Glatzmaier (1999)
Non-uniform stratification

Braginsky (1964)

10% of the shell is stably stratified and 90% unstably
Governing equations

\[ E \left( \frac{\partial V}{\partial t} + (V \cdot \nabla)V - \nabla^2 V \right) + 21_z \times V + \nabla P = \]

\[ = R_a \frac{r}{r_o} T + \frac{1}{P_m} (\nabla \times B) \times B, \]

\[ \frac{\partial B}{\partial t} = \nabla \times (V \times B) + \frac{1}{P_m} \nabla^2 B, \]

\[ \frac{\partial T}{\partial t} + (V \cdot \nabla)T = \frac{1}{P_r} \nabla^2 T + H, \]

\[ \nabla \cdot V = 0, \quad \nabla \cdot B = 0 \]
Boundary conditions

- the non-penetrating and no-slip boundary conditions for the velocity field at the rigid surfaces

- the constant temperature $T_i = 1$ and $T_o = 0$ at the inner (ICB) and outer (CMB) boundaries of the shell, respectively

- the CMB is electrically insulating, while the ICB is electrically conducting
Case $E = 10^{-3}$

$E = 10^{-3}, \ P_m = 3 \ (P_{m_{\text{min}}} = 2.5), \ P_r = 0.2, 1,$

$R_a = 190, 250$

$H = \begin{cases} 
-2.78 & \text{for} \ P_r = 1 \\
-13.9 & \text{for} \ P_r = 0.2 
\end{cases}$

MAG http://www.geodynamics.org/cig/software/packages/geodyn/mag/
\( \tau = 10 \)
MAGNETIC ENERGY

\[
E_m = \begin{cases} 
1 & \text{UNI} \\
0.2 & \text{NON-UNI} 
\end{cases}
\]

\[\tau = 10\]
$B_r$ at $r = 0.85$

\[
\begin{pmatrix}
  P_r = 1 & 190 \text{ UNI} & 250 \text{ UNI} & 190 \text{ NON} - \text{ UNI} & 250 \text{ NON} - \text{ UNI} \\
  P_r = 0.2 & 190 \text{ UNI} & 250 \text{ UNI} & 190 \text{ NON} - \text{ UNI} & 250 \text{ NON} - \text{ UNI}
\end{pmatrix}
\]
$V_r$ at $r = 0.85$

\[
\begin{pmatrix}
P_r = 1 & 190 \text{ UNI} & 250 \text{ UNI} & 190 \text{ NON – UNI} & 250 \text{ NON – UNI} \\
P_r = 0.2 & 190 \text{ UNI} & 250 \text{ UNI} & 190 \text{ NON – UNI} & 250 \text{ NON – UNI}
\end{pmatrix}
\]
\[ B \text{ at } r = r_o \]

\[
\begin{pmatrix}
  P_r = 1 & 250\text{ UNI} & 250\text{ NON – UNI} \\
  P_r = 0.2 & 250\text{ UNI} & 250\text{ NON – UNI}
\end{pmatrix}
\]
$B_r$ and $V_r$ at $r = 0.85$

$P_m = 3, 15, 35$  $P_r = 0.2$

\[
\begin{pmatrix}
B_r & 190\, UNI, P_m = 3 & 190\, UNI, P_m = 15 & 190\, UNI, P_m = 35 \\
V_r & 190\, UNI, P_m = 3 & 190\, UNI, P_m = 15 & 190\, UNI, P_m = 35
\end{pmatrix}
\]
Rossby number

<table>
<thead>
<tr>
<th>$Ra$</th>
<th>$Ro(P_r = 1)/Ro(P_r = 0.2)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>190</td>
<td>0.266</td>
</tr>
<tr>
<td>UNI</td>
<td></td>
</tr>
<tr>
<td>190</td>
<td>0.248</td>
</tr>
<tr>
<td>NON-UNI</td>
<td></td>
</tr>
<tr>
<td>250</td>
<td>0.235</td>
</tr>
<tr>
<td>UNI</td>
<td></td>
</tr>
<tr>
<td>250</td>
<td>0.236</td>
</tr>
<tr>
<td>NON-UNI</td>
<td></td>
</tr>
</tbody>
</table>

$$Ro = \frac{V}{L\Omega} = ER_m P_m^{-1}$$
Conclusions - stratification

- the influence of non-uniform stratification is for our parameters weak

- thin stably stratified region very slightly destabilizes the dynamo for $P_r = 1$, while for $P_r = 0.2$ either does not influence or very slightly stabilizes the dynamo

- we do not observe in our dynamos an eastward drift (Stanley and Mohammadi (2008)) of magnetic flux spots in the equatorial regions, it is always westward

- our stably stratified layer is not very strongly stratified, i.e. our model is perhaps characterized by the mild stable stratification
Case $E = 10^{-4}$

$E = 10^{-4}$, $P_m = 1, 0.75, 0.5$ ($P_{m_{\text{min}}} = 0.45$),

$P_r = 0.2$, $R_a = 850, 1200, 1550$

$H = 0$

PARODY and DMFI  

Aubert, Aurnou and Wicht (2008)
$\tau = 8$

KINETIC ENERGY FOR $E = 10^{-4}$

$P_m = 1$
$P_m = 0.75$
$P_m = 0.5$

$E_k$ vs $R_a$
MAGNETIC ENERGY FOR $E = 10^{-4}$

- $P_m = 1$
- $P_m = 0.75$
- $P_m = 0.5$

$\tau = 8$
\[ B_r \text{ for } R_a = 1550 \text{ and } P_m = 1 \]
$B_r$ for $R_a = 1550$ and $P_m = 0.75$
$B_r$ for $R_a = 1550$ and $P_m = 0.5$
Case $E = 10^{-5}$

$E = 10^{-5}$, $P_m = 0.5, 0.25, 0.1$ ($P_{m_{min}} = 0.08$),

$P_r = 0.2$, $R_a = 8000, 25000$

$H = 0$

PARODY and DMFI  
Aubert, Aurnou and Wicht (2008)
KINETIC ENERGY FOR $E = 10^{-5}$

$E_k$ vs $R_a$

- $P_m = 0.5$ (star)
- $P_m = 0.25$ (triangle)
- $P_m = 0.1$ (circle)

$\tau = 5$
MAGNETIC ENERGY FOR $E = 10^{-5}$

$P_m = 0.5$ ★
$P_m = 0.25$ △
$P_m = 0.1$ ○

$\tau = 5$
$B_r$ for $R_a = 8000$ and $P_m = 0.5$
$B_r$ for $R_a = 8000$ and $P_m = 0.25$
$B_r$ for $R_a = 8000$ and $P_m = 0.1$
## Rossby number

<table>
<thead>
<tr>
<th>$E = 10^{-4}$</th>
<th>$R_a = 1550$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$R_o(P_m = 1)/R_o(P_m = 0.75)$</td>
<td>0.98</td>
</tr>
<tr>
<td>$R_o(P_m = 1)/R_o(P_m = 0.5)$</td>
<td>0.94</td>
</tr>
<tr>
<td>$R_o(P_m = 0.75)/R_o(P_m = 0.5)$</td>
<td>0.97</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>$E = 10^{-5}$</th>
<th>$R_a = 8000$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$R_o(P_m = 0.5)/R_o(P_m = 0.25)$</td>
<td>0.87</td>
</tr>
<tr>
<td>$R_o(P_m = 0.5)/R_o(P_m = 0.1)$</td>
<td>0.71</td>
</tr>
<tr>
<td>$R_o(P_m = 0.25)/R_o(P_m = 0.1)$</td>
<td>0.81</td>
</tr>
</tbody>
</table>
Conclusions - $P_r$ and $P_m$

- for $P_r \geq 1$ and $P_m \geq 1$ the inertia is weak, dynamos are mostly dipolar and large-scale flows are columnar (e.g., Christensen and Aubert (2006))

- for $P_r < 1$ the inertia becomes important but it depends on $P_m$. For $P_r < 1$ and $P_m \geq 1$ (rather $P_m \gg P_{m_{min}}$) is similar to $P_r \geq 1$ and $P_m \geq 1$, i.e. the inertia is again weak and dynamos are mostly dipolar and large-scale flows are columnar, while for $P_r < 1$ and $P_m < 1$ (rather $P_m \leq P_{m_{min}}$) the inertia becomes important and it is possible to observe the breakdown of the columnar structure of the convection in consequence of the dipolar structure breaks generally down. Fluid motion becomes strong in the polar regions and the magnetic field is convected out of polar regions (Sreenivasan and Jones (2006))
• Busse and Simitev (2005) observed at low values of $P_m$ a transition to hemispherical dynamos and at even lower values of $P_m$ a further transition to quadrupolar dynamos (stress-free boundary conditions). These transitions are observed neither in Sreenivasan and Jones (2006) (no-slip boundary conditions, $P_m = P_r = 1, 0.5, 0.2$) nor in our study (no-slip boundary conditions).

• Dipolar dynamo breaks down in Sreenivasan and Jones (2006) for $P_m = P_r = 0.2$ ($P_m < P_{m_{\text{min}}}$), while our magnetic field is dipolar and does not weaken because our $P_m$ is greater or close to the value of $P_{m_{\text{min}}}$. The inertia becomes important but our dynamos remain dipolar.

• It is possible to conclude that $P_m$ governs a measure of inertia for low $P_r$, i.e. a measure of inertia depends on how far we are from $P_{m_{\text{min}}}$.
- the real Earth’s core - $E = 10^{-15}$, $P_m = 10^{-7}$, $P_r = 0.2$, $P_{m_{min}} \approx 2.5 \times 10^{-9}$, i.e. the geodynamo works in the mode $P_m \gg P_{m_{min}}$


THANK YOU

"Do, or do not. There is no 'try'."
- Yoda ("The Empire Strikes Back")

Bavaria...
It's Relentless