Lokálne chyby numerických schém

Peter Moczo Jozef Kristek Martin Galis

Emmanuel Chaljub, Vincent Etienne

FMFI UK Bratislava

GFÚ SAV Bratislava

ISTerre Université Joseph Fourier Grenoble, France

3D numerical schemes					
method		equation formulation	grid	add. specif.	order
FD D CG 2	finite-difference	displacement	conventional		
FD DS PSG 2		displacement -stress	partly staggered		
FD DS SG 2		displacement -stress	staggered		

3D numerical schemes						
method		equation formulation	grid	add. specif.	order
FD D CG 2	finite-difference	displacement	conventional		2
FD DS PSG 2		displacement -stress	partly staggered		
FD DS SG 2		displacement -stress	staggered		
FE L8	finite-element	displacement	conventional	Lobatto 8-point integr.	
FE G1				Gauss 1-point integr.	
FE G8				Gauss 8-point integr.	

3D numerical schemes					
method		**equation formulation**	**grid**	**add. specif.**	**order**
FD D CG 2	finite-difference	displacement	conventional		2
FD DS PSG 2		displacement -stress	partly staggered		
FD DS SG 2		displacement -stress	staggered		
FE L8	finite-element	displacement	conventional	Lobatto 8-point integr.	
FE G1				Gauss 1-point integr.	
FE G8				Gauss 8-point integr.	
DG CF 2	discontinuous Galerkin	displacement	conventional	centered flux	

3D numerical schemes					
method		**equation formulation**	**grid**	**add. specif.**	**order**
FD D CG 2	finite-difference	displacement	conventional		2
FD DS PSG 2		displacement-stress	partly staggered		
FD DS SG 2		displacement-stress	staggered		
FE L8	finite-element	displacement	conventional	Lobatto 8-point integr.	
FE G1				Gauss 1-point integr.	
FE G8				Gauss 8-point integr.	
DG CF 2	discontinuous Galerkin	displacement	conventional	centered flux	
FD D CG 4a	finite-difference	displacement	conventional		4
FD D CG 4b					
FD DS SG 4		displacement-stress	staggered		

3D numerical schemes					
method		equation formulation	grid	add. specif.	order
FD D CG 2	finite-difference	displacement	conventional		2
FD DS PSG 2		displacement-stress	partly staggered		
FD DS SG 2		displacement-stress	staggered		
FE L8	finite-element	displacement	conventional	Lobatto 8-point integr.	
FE G1				Gauss 1-point integr.	
FE G8				Gauss 8-point integr.	
DG CF 2	discontinuous Galerkin	displacement	conventional	centered flux	
FD D CG 4a	finite-difference	displacement	conventional		4
FD D CG 4b					
FD DS SG 4		displacement-stress	staggered		
SE 4 cn, vn	spectral-element	displacement	conventional	GLL integr.	

3D numerical schemes

method		equation formulation	grid	add. specif.	order
FD D CG 2 = FE L8		displacement	conventional		2
FD DS PSG 2	finite-difference	displacement-stress	partly staggered		
FD DS SG 2		displacement-stress	staggered		
FE L8	finite-element	displacement	conventional	Lobatto 8-point integr.	
FE G1				Gauss 1-point integr.	
FE G8				Gauss 8-point integr.	
DG CF 2	discontinuous Galerkin	displacement	conventional	centered flux	
FD D CG 4a	finite-difference	displacement	conventional		4
FD D CG 4b					
FD DS SG 4		displacement-stress	staggered		
SE 4 cn, vn	spectral-element	displacement	conventional	GLL integr.	

3D numerical schemes					
method		**equation formulation**	**grid**	**add. specif.**	**order**
FD D CG 2 = FE L8		displacement	conventional		2
FD DS PSG 2 = FE G1		displacement -stress	partly staggered		
FD DS SG 2		displacement -stress	staggered		
FE L8	finite-element	displacement	conventional	Lobatto 8-point integr.	
FE G1				Gauss 1-point integr.	
FE G8				Gauss 8-point integr.	
DG CF 2	discontinuous Galerkin	displacement	conventional	centered flux	
FD D CG 4a	finite-difference	displacement	conventional		4
FD D CG 4b					
FD DS SG 4		displacement -stress	staggered		
SE 4 cn, vn	spectral-element	displacement	conventional	GLL integr.	

3D numerical schemes					
method		**equation formulation**	**grid**	**add. specif.**	**order**
FD D CG 2 = FE L8		displacement	conventional		2
FD DS PSG 2 = FE G1		displacement -stress	partly staggered		
FD DS SG 2		displacement -stress	staggered		
FE L8	finite-element	displacement	conventional	Lobatto 8-point integr.	
FE G1				Gauss 1-point integr.	
FE G8 = DG CF 2				Gauss 8-point integr.	
DG CF 2	discontinuous Galerkin	displacement	conventional	centered flux	
FD D CG 4a	finite-difference	displacement	conventional		4
FD D CG 4b					
FD DS SG 4		displacement -stress	staggered		
SE 4 cn, vn	spectral-element	displacement	conventional	GLL integr.	

an unbounded homogeneous isotropic elastic medium

and

a uniform cubic grid

all schemes in a unified form:

$$U\left(x, y, z; t+\Delta t\right) = \mathrm{numerical\_scheme}\left\{U\left(t-\Delta t\right), U\left(t\right)\right\}$$

numerical solution in one time step

$$U^{\mathrm{N}}(x, y, z; t + \Delta t) = \text{numerical\_scheme}\left\{ U^{\mathrm{E}}(t - \Delta t),\ U^{\mathrm{E}}(t) \right\}$$

exact values for plane S wave

a relative local error **in amplitude** in one time step

A^{N} = numerical amplitude at $t+\Delta t$

A^{E} = exact amplitude at $t+\Delta t$

$$\varepsilon_{\mathrm{ampl}}^{\mathrm{Rel}} = \left(\frac{\Delta t_{ref}}{\Delta t}\right)^2 \left|\frac{A^{\mathrm{N}} - A^{\mathrm{E}}}{A^{\mathrm{E}}}\right|$$

$\Delta t_{ref} = \Delta t$ for FD DS SG 4

$p = 0.9 \quad s = 1/6 \quad V_P / V_S = 1.42$

a relative local error in the **vector difference** in one time step

U_i^{N} = numerical displacement component at $t+\Delta t$

U_i^{E} = exact displacement component at $t+\Delta t$

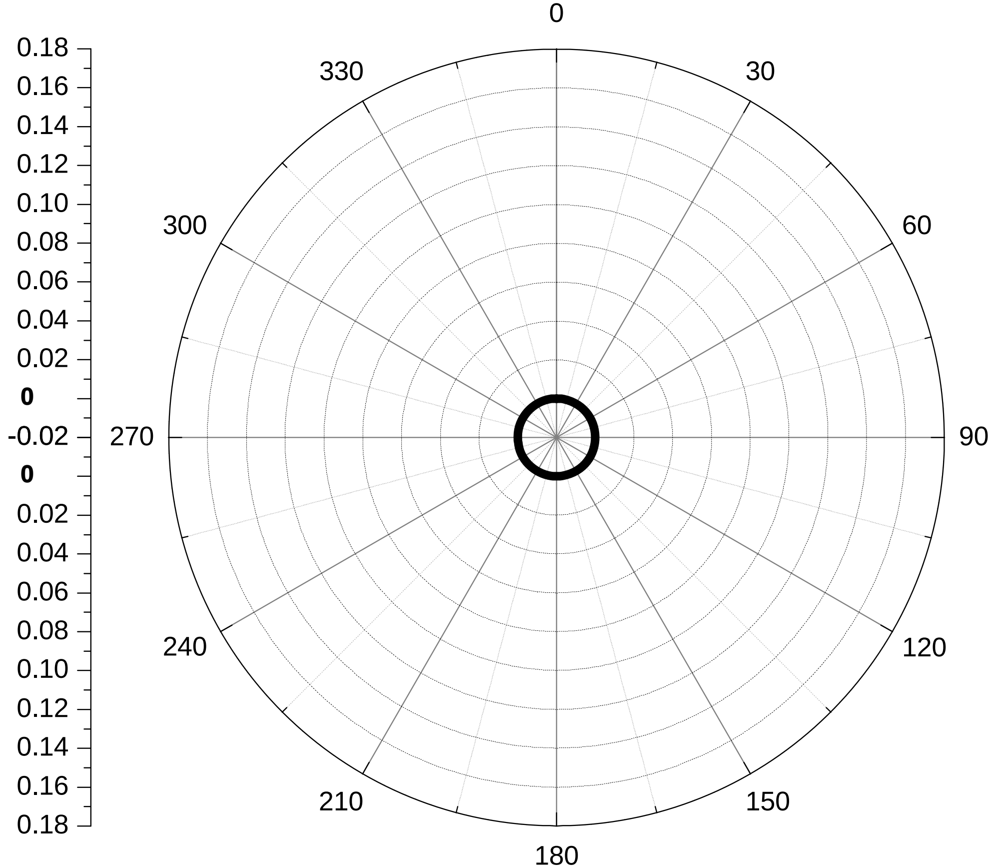
$$\varepsilon_{\mathrm{vdiff}}^{\mathrm{Rel}} = \left(\frac{\Delta t_{ref}}{\Delta t}\right)^2 \frac{1}{A^{\mathrm{E}}} \left[\left(U_x^{\mathrm{N}} - U_x^{\mathrm{E}}\right)^2 + \left(U_y^{\mathrm{N}} - U_y^{\mathrm{E}}\right)^2 + \left(U_z^{\mathrm{N}} - U_z^{\mathrm{E}}\right)^2 \right]^{1/2}$$

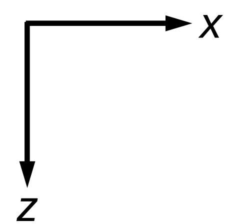
$\Delta t_{ref} = \Delta t$ for FD DS SG 4

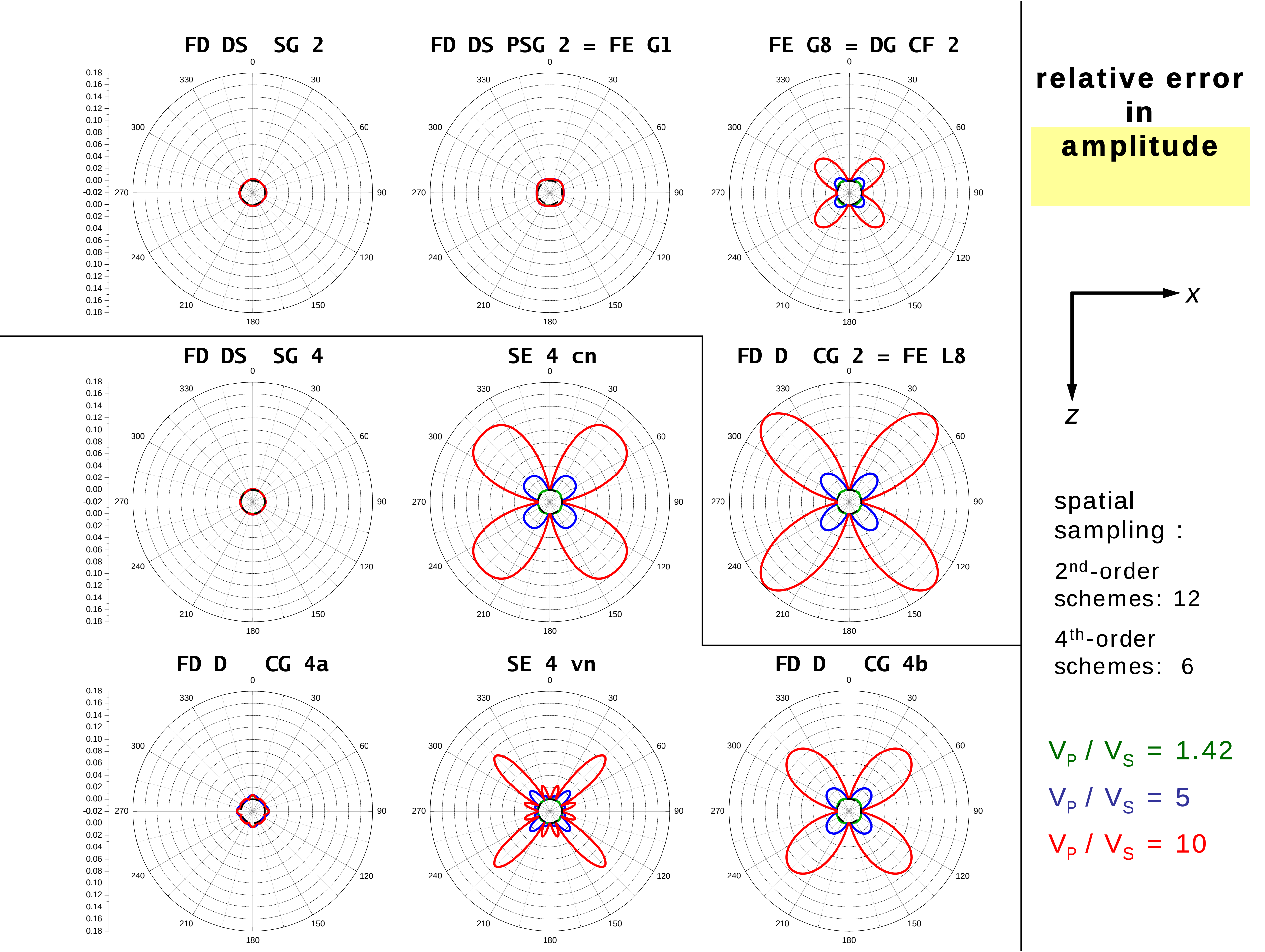
$p = 0.9 \quad s = 1/6 \quad V_P / V_S = 1.42$

solid circle

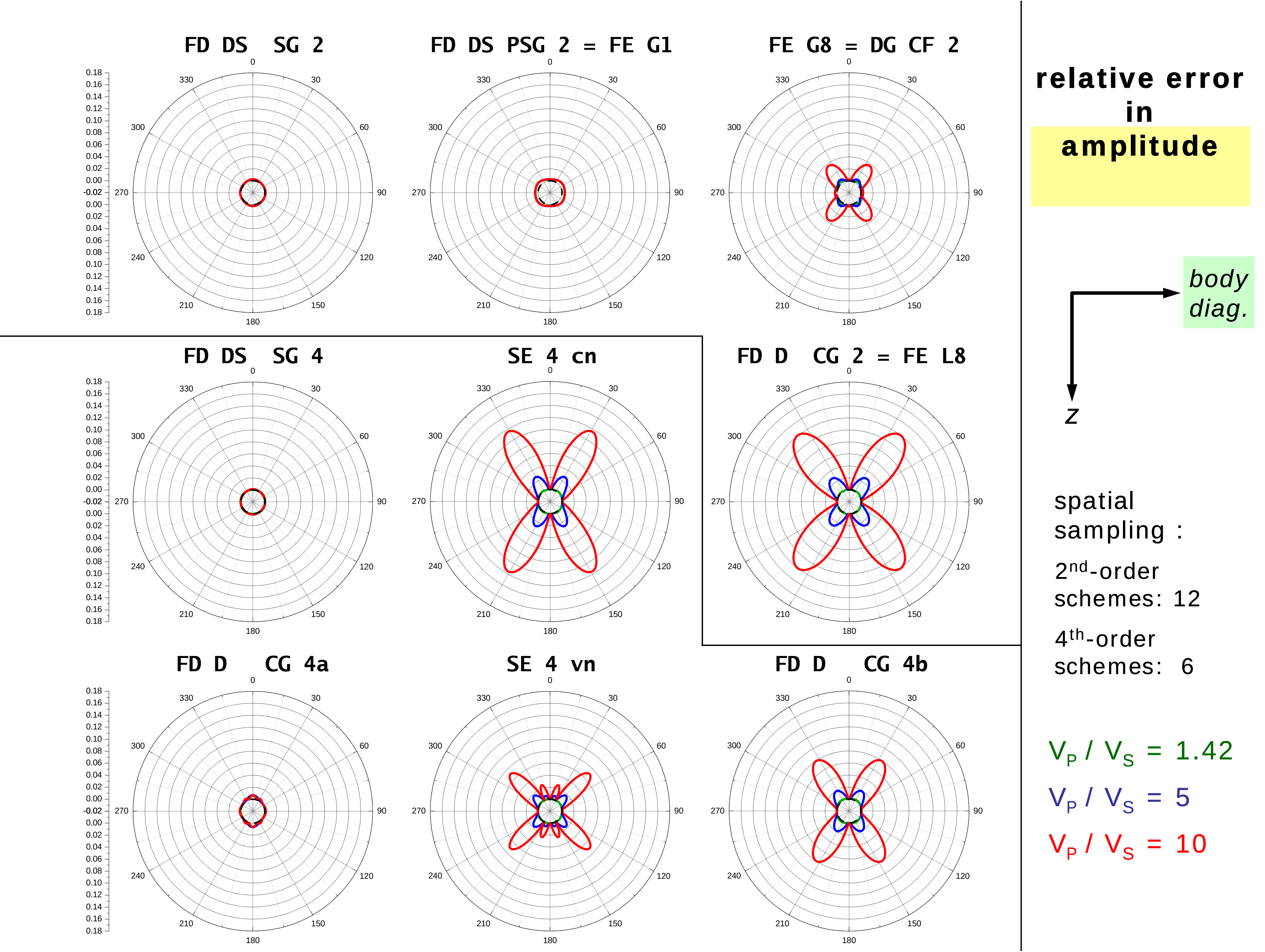
corresponds to zero error

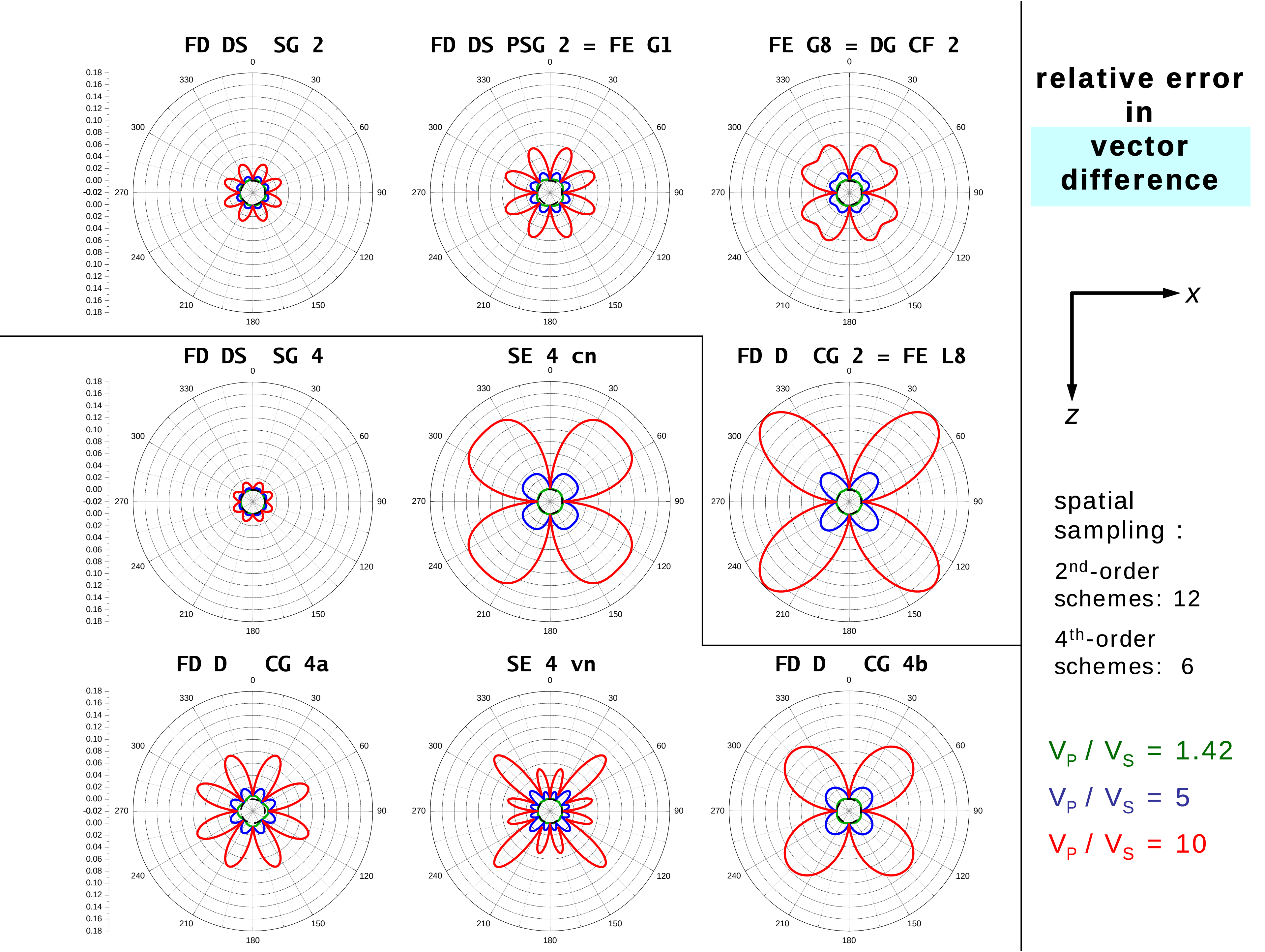


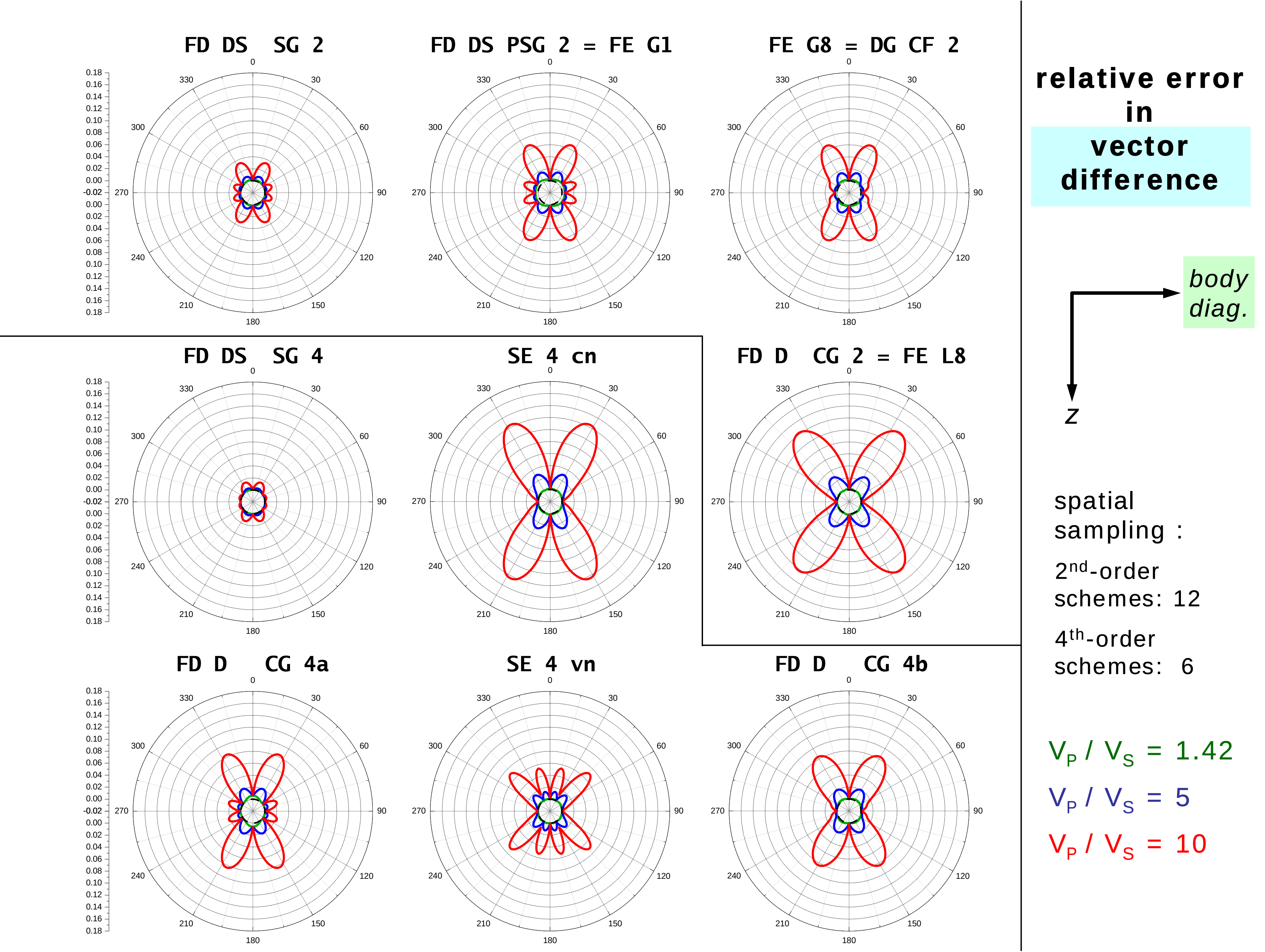


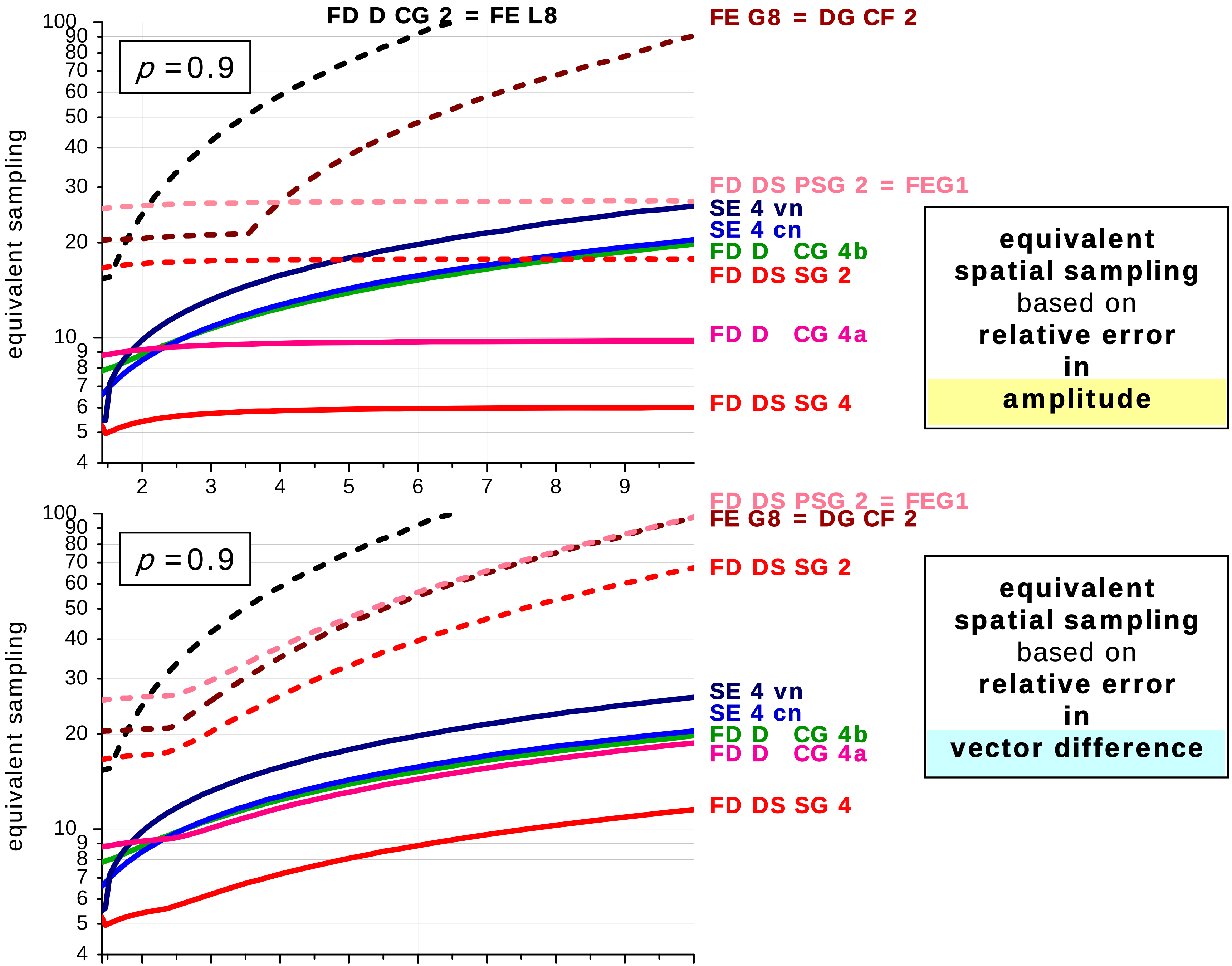


FD DS SG 2
FD DS PSG 2 = FE G1
FE G8 = DG CF 2
FD DS SG 4
SE 4 cn
FD D CG 2 = FE L8
FD D CG 4a
SE 4 vn
FD D CG 4b
relative error in amplitude
x
z
spatial sampling :
2nd-order schemes: 12
4th-order schemes: 6
VP / VS = 1.42
VP / VS = 5
VP / VS = 10


FD DS SG 2
FD DS PSG 2 = FE G1
FE G8 = DG CF 2
FD DS SG 4
SE 4 cn
FD D CG 2 = FE L8
FD D CG 4a
SE 4 vn
FD D CG 4b
relative error in amplitude
body diag.
z
spatial sampling :
2nd-order schemes: 12
4th-order schemes: 6
VP / VS = 1.42
VP / VS = 5
VP / VS = 10


FD DS SG 2
FD DS PSG 2 = FE G1
FE G8 = DG CF 2
FD DS SG 4
SE 4 cn
FD D CG 2 = FE L8
FD D CG 4a
SE 4 vn
FD D CG 4b
relative error in vector difference
x
z
spatial sampling :
2nd-order schemes: 12
4th-order schemes: 6
VP / VS = 1.42
VP / VS = 5
VP / VS = 10


FD DS SG 2
FD DS PSG 2 = FE G1
FE G8 = DG CF 2
FD DS SG 4
SE 4 cn
FD D CG 2 = FE L8
FD D CG 4a
SE 4 vn
FD D CG 4b
relative error in vector difference
body diag.
z
spatial sampling :
2nd-order schemes: 12
4th-order schemes: 6
VP / VS = 1.42
VP / VS = 5
VP / VS = 10



FD D CG 2 = FE L8
FE G8 = DG CF 2
p = 0.9
FD DS PSG 2 = FEG1
SE 4 vn
SE 4 cn
FD D CG 4b
FD DS SG 2
FD D CG 4a
FD DS SG 4
equivalent spatial sampling based on relative error in amplitude
equivalent sampling
100 90 80 70 60 50 40 30 20 10 9 8 7 6 5 4
2 3 4 5 6 7 8 9
FD DS PSG 2 = FEG1
FE G8 = DG CF 2
p = 0.9
FD DS SG 2
SE 4 vn
SE 4 cn
FD D CG 4b
FD D CG 4a
FD DS SG 4
equivalent spatial sampling based on relative error in vector difference
equivalent sampling
100 90 80 70 60 50 40 30 20 10 9 8 7 6 5 4

results and conclusions

the numerical amplitude
is almost independent on the V_P / V_S ratio,
if
the discrete approximations
to the 2nd mixed and non-mixed spatial derivatives have
the same coefficients of the leading terms of truncation errors

this is the case of
FD DS SG 4, FD D CG 4a, FD DS SG 2 and FD DS PSG 2 = FE G1

otherwise
the error in amplitude increases with increasing V_P / V_S ratio

results and conclusions

for all schemes
the error in the vector difference
between the numerical and exact displacements
increases with increasing V_P / V_S ratio

this has to be accounted for by a proper spatial sampling

results and conclusions

FD D CG 2 = FE L8 is the most sensitive
to the increasing V_P / V_S ratio
and for $V_P/V_S > 2$
requires considerably denser spatial sampling
than any other scheme
in order to achieve the same accuracy

the maximum error of
FD DS SG 2, FD DS PSG 2 = FE G1 and FE G8 = DG CF 2
increases in the same way

FD DS PSG 2 = FE G1 and FE G8 = DG CF 2
require denser spatial sampling
than FD DS SG 2
in order to achieve the same accuracy

results and conclusions

the maximum error
of the 4th-order schemes increases in the same way

FD D CG 4a, FD D CG 4b, SE 4 cn and SE 4 vn
require denser spatial sampling
than FD DS SG 4
in order to achieve the same accuracy

the **4th**-order schemes are for $V_P/V_S > 3$
less sensitive to the increasing V_P/V_S ratio
than the **2nd**-order schemes