Local errors of numerical schemes for seismic wave propagation

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A b s t r a c t: We present an analysis of thirteen 3D numerical schemes for modelling seismic wave propagation and earthquake motion for their behavior with a varying P-wave to S-wave speed ratio (VP/VS). The analysis includes 2nd-order schemes - three finite-difference, three finite-element and one discontinuous-Galerkin schemes as well as 4th-order schemes - three finite-difference and two spectral-element schemes. We present all the schemes in a unified form. We assume plane S wave propagation in an unbounded homogeneous isotropic elastic medium. We define relative local errors of the schemes in amplitude and the vector difference in one time step, and normalize them for a unit time. We also define the equivalent spatial sampling ratio as a ratio at which the maximum relative error is equal to the reference maximum error. We present results of the extensive numerical calculations.

We theoretically a) show how a numerical scheme sees the P and S waves if

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the VP/VS ratio increases, b) show the structure of the errors in amplitude and the vector difference, and c) compare the schemes in terms of the truncation errors of the discrete approximations to the second mixed and non-mixed spatial derivatives.

We find four schemes with errors in amplitude almost independent on the VP/VS ratio. We show what distinguishes the finite-difference staggered-grid schemes from the other schemes.

The general theoretical conclusion based on the investigation of the thirteen numerical schemes is that the homogeneity of the approximations to the second mixed and non-mixed spatial derivatives in terms of the coefficients of the leading terms of their truncation errors as well as the absolute values of the coefficients are key factors for the behaviour of the numerical schemes with increasing VP/VS ratio.

The practical conclusion for the existing numerical schemes is that the dependence of the errors in the vector difference on the VP/VS ratio should be accounted for by a proper (sufficiently dense) spatial sampling. We quantified the proper sampling with respect to the local errors in amplitude and in the vector difference.

Key words: numerical schemes, numerical approximations, numerical analysis, computational seismology, theoretical seismology